Accidental Wow Defect Evaluation Using Sinusoidal Analysis Enhanced by Artificial Neural Networks

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Abstract. A method for evaluation of parasitic frequency modulation (wow) in archival audio is presented. The proposed approach utilizes sinusoidal components as their variations are highly correlated with the distortion variations. The sinusoidal components are extracted from audio signal by means of sinusoidal modeling procedures being often severely distorted and in case of wow also significantly modulated. The algorithm for sinusoidal component evaluation utilizes both magnitude and phase spectra information to enhance the tracking process. Additionally, a neural-network based prediction module is proposed to improve the tracking abilities in case of component discontinuities. Experiments concerning prediction of tonal component's values are performed revealing that prediction can enhance sinusoidal modeling of wow distorted signals effectively.

1 Introduction

Wow defect is a distortion defined as parasitic frequency modulation and it is perceived as pitch fluctuation of audio program. It is introduced into audio by motor speed fluctuations, tape damages and inappropriate editing techniques [1]. As wow leads to undesirable variations of all frequency components in distorted sound, the most straightforward approach is to track a particular component to estimate the parasitic modulation.

The analysis of various frequency components was found to be applicable for wow evaluation. These can be genuine sound tonal components [2,3] as well as artifact components such as hum or bias [4,5]. The presented algorithm concerns the first situation in which tonal components are employed for wow defect estimation. This approach is based on sinusoidal modeling, since it is assumed that at least few salient components can be found in the distorted signal. The presented method is dedicated for accidental wow evaluation, which unlike pseudo-periodic

wow found in gramophone recordings [6,7], is of short duration but of strong variations, simultaneously.
Since the audio program pitch is constant, the short duration of accidental wow allows often employing a single tonal component if such a component is present,. More often a set of tonal components must be used for wow evaluation. In this situation a median-based procedure is applied to the extracted tonal components in order to indicate the most likely variations. This is, however, a non-trivial tasks, because a very small differences between estimated and genuine modulation waveform result in audible defects of the restored signal. Thus it is essential to maximize the number of validly extracted tonal components.
In this paper, the algorithm for wow evaluation based on magnitude and phase spectrum is presented. Also experiments concerning prediction of tonal component's values are performed, since it is assumed that prediction can enhance sinusoidal modeling of wow distorted signals effectively.

2 Sinusoidal Modeling of Polyphonic Sounds

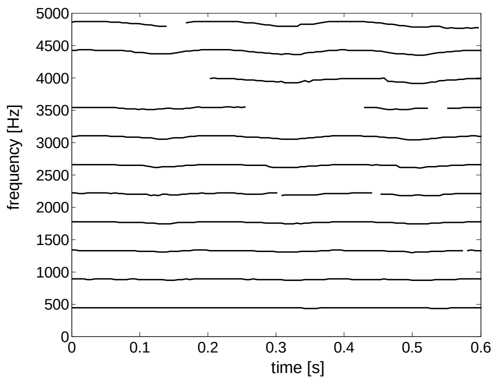
The variations of tonal components strongly correspond to variations of parasitic modulation waveform thus wow estimation can be performed by means of tonal component analysis. The very common approach for tonal analysis of audio signals is sinusoidal modeling. This approach, based in Fourier Theorem, expresses the audio signal as a sum of sinusoidal components having slowly-varying frequencies and amplitudes. For audio signal the following relation can be shown:

$$x(t) = \sum_{p=1}^{P} a_p(t) cos\big(\phi_p(t)\big). \tag{1}$$

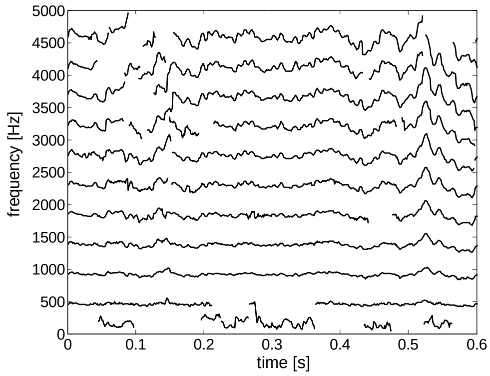
$$\phi_p(t) = \phi_p(0) + 2\pi \int_0^t f_p(u) du. \tag{2}$$

where P corresponds to number of sinusoidal components (partials). The parameters a_p and f_p correspond to amplitude and frequency values of partial. The successive values of f_p ,which create the frequency track, also called MQ track, are processed to obtain the wow modulation pattern called Pitch Variation Curve (PVC).
Sinusoidal modeling, initially used in additive synthesis [8], was found useful in numerous applications including audio sound restoration [9]. However, the original purpose of sinusoidal modeling was the analysis of simple sounds (musical instruments, speech), fitting to the assumption expressed by (1), and distorted archival sounds, which are of interest here, having more complex tonal structure. The modeling of such sounds signals is at least less effective and surely more difficult, since they are likely to be contaminated by several distortions e.g., noise, clicks, which are represented by non-sinusoidal components. Moreover, the wow

distortion itself introduces frequency modulation to the tonal components, which if strong, makes the analysis very difficult. Figure 1 shows the tonal components of a clarinet sound and of a wow-distorted sound (archival sound example). It can be noticed that tracks of a clarinet sound are smooth while tracks of wow distorted sound are rough.



(a) Frequency tracks from a clarinet sound.



(b) Distorted frequency tracks from a wow-contaminated sound.

Fig. 1: Frequency tracks evaluated by means of sinusoidal modeling.

Different attempts have been taken to enhance sinusoidal modeling of polyphonic and non-stationary signals. The main difference between modeling of monophonic and polyphonic signals is that the trajectories of stationary monophonic signals (steady-state sound) can be formed when only frame-to-frame information is used. In case of non-stationary monophonic (e.g. having emphvibrato, *portamento*) and polyphonic signals the trajectories formation should be

performed on the basis of a number of adjacent time-frames analysis. Such an approach, proposed in [10] utilizes the analysis of future possible trajectories of a particular track. The optimal trajectory is assumed to be the track continuation. The approach employed Hidden Markov Models to deter-mine the best trajectory.
Another approach was proposed by Lagrange at all [11]. It also analyses the future trajectories but those ones created by predicted values. Linear prediction (LP) is used forecast the succeeding values of a frequency track. It is showed that this kind of processing can effectively enhance the tracking of modulated tonal components (having *vibrato*). Sinusoidal modeling enhanced by linear prediction was also used for sound gap restoration [12,13].
This approach appears to be valid also for modeling of wow-distorted signals. However, as already shown in Fig. 1, tonal components of wow-distorted sounds are of non-linear nature. In such a situation the non-linear modeling should be performed, thus neural-networks-based prediction is proposed instead of linear prediction.

3 Neural Networks

3.1 Multi-Layer Perceptron

Time series forecasting is one of the most popular usage of the artificial neural networks (*ANN*). Neural networks (*NN*) prediction applications date back to 1964 and from that moment were commonly used in different scientific researches and practical applications (see [14] for comprehensive overview). The NN are self-adaptive methods with many benefits comparing to standard and well known prediction techniques such as the linear prediction. Main NN advantages arise from few priori assumption needed about the input data as well as abilities to generalize and model nonlinearities. The later factor is especially important to this research as the MQ trajectories with wow defect are highly nonlinear, which can be noticed in Fig. 1. Different ANN's were used to forecast time series. Among many others the multi-layer perceptrons (*MLP*) which are feedforward networks seem to be the most popular [14,15,16,17,18,19]. Thus, the MLP was used in the described experiments. A typical MLP structure is given in the Fig 2. The MLP structure is defined by several variables. The key factors are the numbers of inputs, outputs and hidden nodes. Also other determinants such as the activation functions, the nodes interconnections, the training procedure and the input data normalization play important role in MLP-based forecasting. Detailed reasoning behind the chosen MLP structure used in prediction measurements as well as the experiments outline is given in Section 5.

3.2 Error Measures

Different error measures can be applied as the forecasting performance indicators (see [14] for overview). Each of them has some advantages and limitations

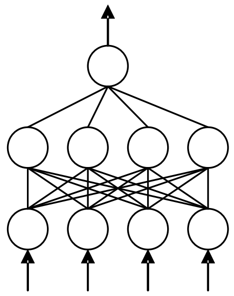


Fig. 2: Typical multi-layer perceptron MLP.

and there is no common agreement which one is appropriate for most forecasting situations. As both prediction methods utilized the same data in the performed experiments the mean square error (MSE, defined by (3)) was computed allowing for direct comparison of both methods.

$$MSE = \frac{\sum e_t^2}{N}. \tag{3}$$

where, e_t is an individual forecast error, N is a number of error terms.
Additionally, the median absolute percentage error ($MdAPE$, defined by (4)) was evaluated to assess the relative forecasting accuracy of both methods:

$$MdAPE = median\big(\mid e_t \mid \big). \tag{4}$$

where, e_t is an individual forecast error.
Both error measures were recommended by Gardner [20] and used by Zhang, Patuwo and Hu in their comprehensive study of MLPs prediction abilities [19].

4 The Algorithm for Wow Evaluation

Wow modulation pattern is obtained in two stages. In the first stage, the sinusoidal modeling is performed which takes as an input the distorted sound and as an output the frequency tracks (frequency values). The block-diagram of this stage is presented in Fig. 3. The second stage computes PVC from the evaluated frequency tracks.

An input signal is divided into analysis frames (time-frames) by means of windowing. The Hamming window is used in order to achieve a good main-lobe to side-lobe rejection ratio. The zerophase windowing is performed to remove linear trend from phase spectrum [8]. The analysis frame is also zeropadded to improve frequency resolution. DFT of each analysis frame is computed to obtain spectrogram representation. Next, candidates for tonal components are evaluated as meaningful peaks of magnitude spectrum according to the following formula:

$$X_m(k-1) < X_m(k) \qquad \wedge \qquad X_m(k+1) < X_m(k). \tag{5}$$

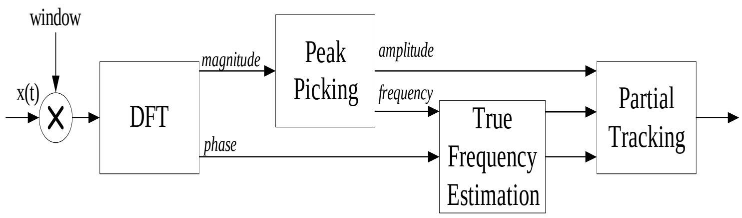


Fig. 3: Block diagram of sinusoidal modeling approach for wow evaluation.

The instantaneous *"true"* frequencies of evaluated peaks are determined be means of frequency reassignment (6)[21]. This is fairly essential since a certain error is introduced to frequency estimation by DFT. The frequency reassignment method as-signs a value of frequency to center of gravity of each bin instead of geometrical center [22]:

$$\hat{f}_0 = k\frac{F_s}{N} + \Im\left\{\frac{X_{w'}(k)}{X_w(k)}\right\}\frac{Fs}{2\pi}. \tag{6}$$

where $X_w(k)$ and $X_{w'}(k)$ are DFT spectra using window and its derivative.
The last step of sinusoidal modeling is frequency track formation. Firstly, the evaluated peaks are sorted due to their magnitude values in order to select the most salient components. Unlike in usual additive synthesis processing, only the most salient components are extracted since they are assumed to depict the wow defect most reliably. Secondly the peaks are linked to existing trajectories in a frame-to-frame processing [23]:

$$\left|f_k^{i-1} - f_l^i\right| < \Delta_f. \tag{7}$$

where, f_k^{i-1} is the frequency of the processed track in frame $i-1$ and f_l^i is the frequency of matched peak in frame i. The parameter Δ_f (frequency deviation) is the maximum frequency distance between track and its continuation.
This criterion allows matching peaks to the trajectories however it does not validate the selected continuation. Magnitude values can be also used to enhance the tracking however the amplitude trajectory was found to have irregular patterns. It is propose to utilize also phase spectrum information to validate the continuation.
For the analyzed partial having phase value ϕ_k^{i-1} in frame $i-1$, the phase value predicted in frame i is equal to:

$$\hat{\phi_k^i} = \phi_k^{i-1} + \frac{R}{N_{DFT}}. \tag{8}$$

where, N_{DFT} is the length of the zeropadded DFT and R is the frame hop distance. The error of phase prediction can be evaluated as:

$$\phi_{err} = \left|\hat{\phi_k^i} - \phi_l^i\right|. \tag{9}$$

The prediction error ϕ_{err} is in the range $[0;\pi)$. The value near 0 suggests phase coherence in two adjacent frames. Otherwise, if value of error is near π, it may indicate phase incoherence and invalid track continuation (see Sect. 4).
The presented algorithm for partial tracking employs frequency matching criterion (7) for continuity selection and phase prediction error for the validation of that continuity. The track termination is associated with a high value of phase prediction error, being different from the original frequency deviation condition (8).
After the frequency tracks are evaluated PVC is computed. Firstly, tracks are normalized in a way that ensures the value 1 for the parts of analyzed signals which are not distorted. It is assumed that the first few frames of input signal are not distorted. The normalization of tracks values to relative values is performed in the following manner:

$$RF_k(i) = \frac{f_k^i}{f_k^1}. \tag{10}$$

where f_k^i is the value of frequency track in the frame i ,and f_k^1 is the value of frequency track in the first frame.
It was found sufficient to utilize only one frequency track for PVC generation under condition that this track has valid values throughout the whole selected region. More often, however, PVC must be evaluated on the basis of a few tracks, The median was found to provide a satisfactory PVC evaluation for this purpose [1]. The median- based PVC is evaluated as follows:

$$PVC_{median}(i) = median\big(RF_k(i)\big). \tag{11}$$

5 Experiments

5.1 Sinusoidal Modeling of Wow Distorted Sounds

The experiments were carried in two steps. Firstly, the tracks were evaluated by means of the algorithm presented in Sect. 4. In the second step the linear-based-prediction and neural-networks-based prediction applied to frequency tracks are com-pared. The archival sound example from the Polish National Film Library and from the Documentary and Feature Film Studio was chosen. The spectrogram and evaluated tracks are depicted in Fig. 4.
It can be noticed that some tracks are distorted and the tracks having high frequency values are also erroneously linked due to strong parasitic modulation. Although the PVC computation based on the detected tracks is valid (Fig. 5) it is strongly desired to improve the tracking. The first problem concerns the evaluated tracks which genuinely belong to one component. This situation is well depicted by the tonal component having its values around 1800 Hz (Fig. 5). The preferred situation would be that this component is evaluated as a one frequency track. It is, however, non-trivial task since the component is visibly distorted. Another problems concerns the erroneous tracking when a strong parasitic modulation occurs, especially for high-frequency components (see components at the

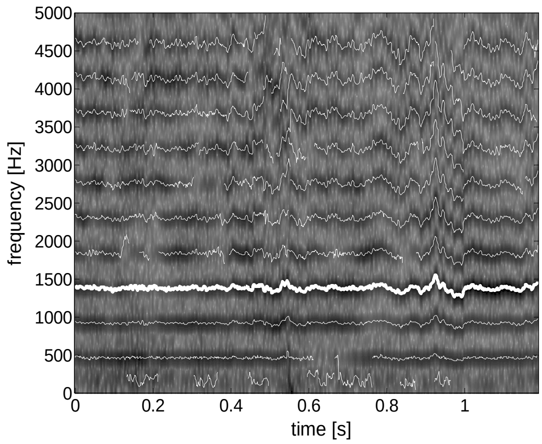


Fig. 4: Tracks evaluated by the algorithm. The track depicted by a bold line is used in further experiments.

1 second in Fig. 4). The tracks variations are so high that frame-to-frame processing does not allow valid tracking.
To overcome mentioned problems an experiment concerning prediction of future track value is performed.

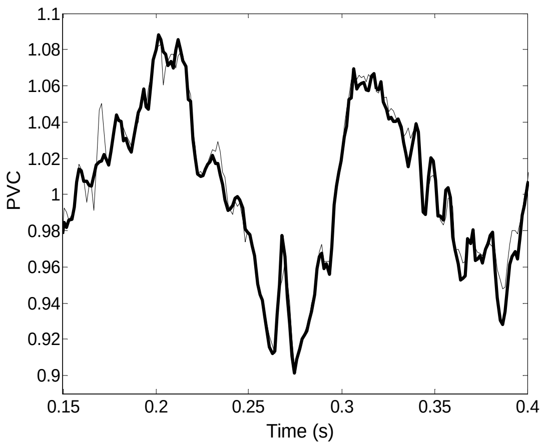


Fig. 5: Fragment of PVC courses. Solid line corresponds to PVC evaluated from single track. Solid thin line corresponds to PVC evaluated from a set of tracks.

5.2 Experiments Concerning Prediction of Tracks Future Values

In order to examine the LP and NN performance in the prediction of the MQ trajectories two simulated experiments were performed. Firstly the LP-based forecast were computed and then compared with results of the NN-based predictions. In both prediction methods the numbers of inputs and prediction algorithms complexity were selected as experimental factors. The experiments were conducted for one-sample-ahead forecasting.
In both experiments some real MQ trajectories obtained from archival sound tracks were selected as the input data. All of the tracks were preprocessed to better fit the inputs of the prediction methods. Firstly an external normalization was performed. The input data were normalized to the range from -0.9 to 0.9 according to equation (12).

$$x_o = 1.8 \frac{(x_n - x_{min})}{x_{max} - x_{min}} - 0.9 \quad (12)$$

where, x_n is a input data sample, x_{min} is a minimal value in the input data set, x_{max} is a maximal value in the input data set.
After the linear operation the statistical normalization which is given by equation (13) was performed.

$$x_o = \frac{x_n - \bar{x}}{s} \quad (13)$$

where, x_n is an input data sample, $\bar{x}$ is a mean value of the input data set and s is a standard deviation of the input data set.
At the end the input vectors were smoothed using a 3-rd order moving average zero-phase filter. The MQ trajectory is given in Fig. 6 (depicted as bold white line) the one from Fig. 4 processed in such a manner.

5.3 Linear Prediction

The LP abilities to forecasts MQ trajectories were tested on the same sample set as the NNs. A sliding-time-window technique was used. In each time window the prediction filter was build and one sample was forecasted. The prediction involved a different combination of LP parameters. The subject of examining were: the algorithm order (LP order) which defines the number of the prediction filter coefficients, and the LP length which is the number of past samples used to find the coefficients. The autocorrelation method of autoregressive (AR) modeling was used to compute filter coefficients. The obtained results are given in It can be noticed from Tab. 1 that the key factor influencing the prediction performance is the LP length, i.e., the prediction error decreases with greater lengths. The LP order plays less important role and according to obtained results a greater number of LP coefficients can trigger a higher prediction errors. It is probably due to the chaotic nature of the MQ trajectories, whereas, the LP tries to model it as a linear process.

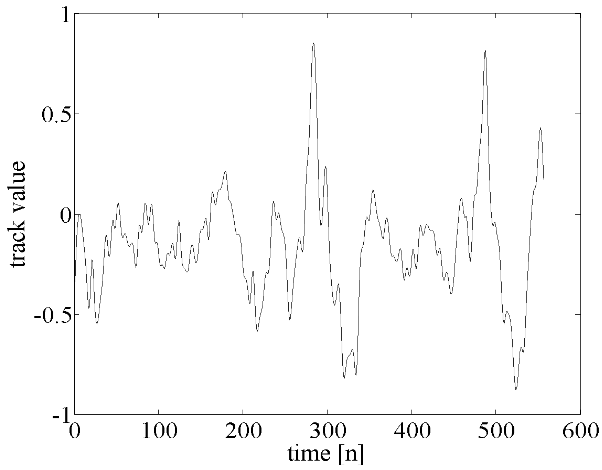


Fig. 6: A preprocessed frequency track.

Table 1: LP-based Prediction Error

LP length	**LP order**				**Error Measure**
4	0.0116				MSE
	0.3156				MdAPE
8	0.0045	0.0052			MSE
	0.1838	0.2110			MdAPE
16	0.0029	0.0031	0.0037		MSE
	0.1491	0.1512	0.1682		MdAPE
32	0.0020	0.0021	0.0022	0.0025	MSE
	0.1226	0.1247	0.1275	0.1365	MdAPE

5.4 NN-Based Prediction

One of the most popular ANNs used for the time series forecasting are the multi-layer perceptrons (MLP) [[14,15,16,17,18,19]]. Several factors constitute the overall MLP-network architecture. The key factors are the number of layers, the number of nodes in each layer, and the neuron's activation functions. Also the learning algorithm as well as the training and validating sets can play some role on prediction results.
Several studies give some practical guidelines for choosing the NN architecture, however, a systematic approach to this problem is not known to the authors. Some theoretical works showed that a single hidden layer is sufficient for ANN to approximate any nonlinear function with a desired accuracy [24,25]. It is also the one-hidden-layer MLP which was most commonly used in the forecasting applications so far [14]. Thus only one hidden layer was used in the conducted experiment. Considering the number of input and hidden nodes resent findings of Zhang, Patuwo and Hu [19] showed that in case of the MLP-based one-sample-ahead forecasting greater attention should be put on the input's number. Therefore in the presented experiment only the input layer size was varied. The number of hidden-layer nodes was equal to current inputs number. The input layer sizes were chosen to allow comparison with the LP-based prediction method, and were set to 2, 4, 8, 16 and 32 samples. The neuron-activation functions were chosen following the standard convention following by most of the authors [14]. The logistic functions were used for hidden nodes and linear function for the output node.
The training set (in-sample data) and testing set (out-of-sample) were build of some real MQ trajectories which were processed according to normalization and smoothing procedures given earlier on (see (11) (12) and the following paragraph). As the in-sample data the 80 % of the trajectory values were used. The remaining 20 % constituted the out-of-sample set.
The Levenberg-Marquardt backpropagation was employed as the training algorithm. This method was found out to converge faster then the standard gradient descent with momentum and adaptive learning rate backpropagation. Both the goal MSE value and the minimum performance gradient were set to 0.001. The maximal number of epochs was set to 1000.
The sliding windowing technique was used in the NN performance evaluation. The utilized time window was build of data from the out-of-sample set. After each prediction the network's weights and biases were changed allowing for the NN adaptation. Experiments on each MLP structure were repeated 200 times with randomly initiated weights and biases. The overall MSE was computed as a mean values along all of the individual MSEs. The overall MdAPE, on the other hand, was computed as the median value of the local MdAPEs. The obtained results are given in Tab. 2.
It can be noticed from the Tab. 2 that with the increasing number of the input nodes the prediction error decreases. However, after reaching the point of 16 inputs it grows up again. It is probably due to the under-fitting in the learning stage as the 32-32-1 structure is quite large and a lower goal MSE should be

Table 2: NN-based Prediction Error

MLP Structures					**Error Measure**
2-2-1	4-4-1	8-8-1	16-16-1	32-32-1	
0.0086	0.0018	0.0009	0.0009	0.0035	MSE
0.0898	0.0777	0.0706	0.0679	0.0817	MdAPE

used here. Yet the most important observation is that the NN performance is better then the LP-based forecasting (see Tab. 1). Only the MSE for the simplest MLP structure is greater then the LP prediction error. All the other error measurements indicate lower values for the NN-based forecasting.

6 Conclusions

The conducted experiments aimed at showing that a prediction module can enhance the sinusoidal modeling (that was also showed in some cited papers), however the experiments were focused rather on neural-networks-based prediction, since LP-based prediction was assumed to not fit the non-linear model of wow-distorted tracks. The main conclusion which can be drawn from the forecasting experiments is that the NNs outperforms the simple LP methods used so far for the perdition of the future values of the MQ trajectories. These findings are consistent with general knowledge about the NN abilities to model and to forecast non-linear functions. Further research is needed, however, to determine the most appropriate MLP structure for the MQ forecasting task. Also, a greater attention must be put on the preprocessing stage as it can influence the obtained results. The final conclusion is that even at the present stage of research, the intelligent approach to parasitic modulation in audio brought some interesting and practically justified results.

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